International Journal of Theoretical Physics, Vol. 30, No. 5, 1991

Gravitational Coupling at the Planck Scale

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Received October 2, 1990

We consider the possibility that gravitational interaction could have originated in the decoupling of matter from antimatter at the Planck scale.

1. INTRODUCTION

We shall first briefly review how the notion of gravitational interaction took shape over the years.

The Einstein gedanken experiment according to which accelerated observers (e.g., observers in an elevator) can attend effects imputable to the presence of a gravitational source was a first attempt to introduce the gravitational interaction from a Newtonian concept: acceleration. Such effects (i.e., associated to accelerated trajectories) were broadly classified as inertial effects.

The idea that a relationship might exist between gravity and inertia was further expressed via the identification of the Newtonian concepts of inertial and gravitational masses m_i and m_g. The notion of gravitational coupling thus emerges from a reformulation of Newton's law:

$$\Gamma \equiv \frac{d^2r}{dt^2} = \frac{m_g}{m_i} \nabla U$$

where m_g/m_i is a universal constant. The (experimentally confirmed) identification $m_i = m_g$ clarifies the principle of equal accelerations. There is also a hint that gauge fields (e.g., ∇U) could be mediators of the interaction. General relativity then provided a covariant formulation of the principle of equivalence (equal accelerations) imposing that test particles should follow geodesics of the (curved) space-time geometry: an extremal principle.

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In this work, we would like to further question the nature of the gravitational interaction and of its mediation, and will show that such principles can be derived from quantum considerations. Note that the introduction of the gravitational coupling strength also brought the problem of its comparison with others and of the unification of interactions. The situation is best illustrated by electromagnetism. Here the Coulomb law of electrostatics

$$F = e_1 e_2 / r^2$$

displays a striking analogy with Newton's law of gravitation

$$F = m_1 m_2 / r^2$$

and underlines the role of charge ratios in the unification of coupling strengths ($m/e \sim 1/\sqrt{G}$, where G is Newton's coupling contant). We shall focus on such ratios. In particular we shall see that the ratio M/M^* (mass over magnetic mass), which could be viewed as a covariant generalization of m_g/m_i, plays a role in the description of gravitational interaction as a causal phenomenon (no instantaneous propagation).

This will lead us to propose that gravity could be a manifestation of the decoupling of matter from antimatter at the Planck scale, via acausality breaking. More broadly one might conjecture that a decoupling of our universe from an anti-universe (mirror universes) could provide a global setup for the onset of gravity as a nonrepulsive process. We shall see that stringlike objects could have mediated the process, thus possibly turning the initial singularity around.

2. QUANTUM ONSET OF CURVATURE AND HORIZONS

Our purpose here is to set a relationship between space-time curvature and an inertial (and quantum) process.

As an illustration, we shall place ourselves in the Rindler wedge of a flat Minkowski space with metric $ds^2 = \eta_{ab}\, dx^a\, dx^b$. Inertial effects associated with accelerated trajectories are well known, and for simplicity we focus on constant-acceleration trajectories $x^a x_a = a^2$: pair creation and thermal effects take place along such trajectories. We shall thus assume that a spontaneously created particle can be suitably described by an eight-dimensional phase space, where the metric is, at the Planck scale, and to first order, affected by the acceleration of the particle only:

$$d\tau^2 = g_{AB}\, dx^A\, dx^B = \eta_{ab}\, dx^a\, dx^b + (m\hbar)^2 \eta_{ab}\, d\dot{x}^a\, d\dot{x}^b$$

where m is the mass of the particle. The flat metric of the configuration space is thus recovered in the classical limit as $\hbar \to 0$. Assuming that the quantum particle is not localized, the embedding of its configuration space into phase space is described by

$$X^A = X^A(\xi^b)$$

where ξ^b are coordinates on the Rindler wedge. Thus, configuration space acquires curvature as a submanifold of the (curved) phase space (and via quantum correction). The viewpoint here is not that, on a given curved space-time, the concept of particle and field quantum could be observer dependent. Rather, the space-time curvature itself is provided a quantum and observer-dependent correction. No field equation so far has been imposed, but we shall come back to this point. Assuming that constant-acceleration trajectories in the Rindler wedge are parametrized by their proper time s, one has

$$x = a \text{ ch } s/a, \qquad t = a \text{ sh } s/a$$

$$\dot{x} = \text{sh } s/a, \qquad \dot{t} = \text{ch } s/a$$

which implies

$$d\tau^2 = g_{AB}\, dx^A\, dx^B = ds^2\left(1 - \frac{m^2\hbar^2}{a^2}\right) + (da)\omega$$

where $\omega(ds, da)$ is a one-form.

The Rindler wedge is thus provided a Killing horizon (the constant-acceleration hyperbola $a = m\hbar$) and curvature $R = 2[1/(a^2 - m^2\hbar^2)]^2$. This suggests that a Schwarzschild-type horizon could be recovered, in this context, as an asymptote to horizons of the type $a = m\hbar$. This also suggests that curved solutions to Einstein's vacuum equation could be viewed as limits (asymptotes) of metrics such as the above one, after a quantum correction has been imposed to arbitrary order:

$$d\tau^2 = ds^2\left(1 - \sum_{k \geq 0} \frac{(m^2\hbar^2)^k}{a_k^2}\right) + \text{etc.}$$

or more generally

$$d\tau^2 = ds^2\left[1 - \int_0^\infty (m\hbar)^{2a} d\left(\frac{1}{a}\right)\right] + \text{etc.}$$

In this setup, curvature finds its origin in the quantum vacuum.

For later purposes, we underline the possibility of a quantum onset of two causal (respectively future and past-oriented) disconnected and curved asymptotic regions on the Rindler wedge. The process finds its origin in the decoupling of spontaneously created pairs of particles–antiparticles: the horizon is a manifestation of their decoupling, and curvature a manifestation of their gravitational correlation.

In the next section we proceed further, focusing on accelerated trajectories which are more canonical from the cosmological viewpoint, i.e., related to the expansion rate (acceleration) of the universe.

3. GRAVITATIONAL STRINGS AND ACAUSAL COUPLING

If gravity and curvature are to find their origin within particle production and inertial processes, so should the gravitational coupling strength. We shall consider this matter here.

On one hand we would like to take into account features inherent to the structure of the gravitational interaction: its long range, and this basic fact, namely that it carves the large-scale structure of the universe. This leads us to focus on canonical accelerated trajectories, i.e., those which govern the expansion rate of the FRW model: the conformal isometries. Note that already at the classical level, there is a (large-scale) relationship between the expansion rate $a(t)$ and the matter density $\rho(t)$ in the universe, clearly expressed by the Hubble law:

$$\left|\frac{\dot{a}(t)}{a(t)}\right|^2 = \frac{8}{3}\pi G\rho(t) - \frac{c^2}{R^2 a^2(t)}$$

On the other hand, we would like to take into account similarities which are inherent to the gravitational and electromagnetic couplings. These are particularly striking in the presence of Dirac string singularities, the so-called magnetic monopoles. In the presence of such charges, it is the small-scale structure which is expected to be affected. We first consider this issue.

Let us recall the following experimental result (Mikhailov, 1983). While accelerating ferromagnetic aerosols for the purpose of detecting particles displaying features of the magnetic charges, Mikhailov established an empirical value of the ratio of the magnetic charge to the electric charge: an integer (n) multiple of the Sommerfeld fine structure constant ($\alpha = 1/137$). A theoretical explanation of this quantization rule has been proposed (Magnon, 1986, 1988, 1989). Recall that a Dirac monopole source can be viewed, geometrically, as a point source (e.g., the origin) deleted from $\mathbf{R}^3$, which thus acquires an $\mathbf{R}^3 - \{0\}$ topology. The resulting (flat) space-time structure is a nontrivial $U(1)$ bundle over this noncontractible base space. The charge of the monopole is a measurement of the integer number of twists of the bundle around its $U(1)$ fiber. Unlike the electric charge $\int_{S_2} {}^*F_{nb}\, dS^{ab}$ (where S_2 is a 2-sphere surrounding the monopole), the magnetic charge $\int_{S_2} F_{ab}\, dS^{ab}$ is topological and quantized, and is known as the first Chern class of the $U(1)$ bundle.

Equivalently, the gauge potential A_b ($F_{ab} = \nabla_{[a}A_{b]}$) displays a discontinuity on any S_2 surrounding the source. This fictitious point spans a two-dimensional world-sheet (the Dirac string singularity). In other words, the field strength F_{ab} is a closed, not globally exact 2-form which must be viewed as cohomologically dual to the noncontractible 2-spheres (or 2-chains) surrounding the fictitious (deleted) source. Clearly, these geometrical features are controlled by α, which appears as a universal coupling constant.

Note also that in the complex regime, the constancy of the ratio of electric to magnetic charge is guaranteed by the existence of self-dual solutions to the field equations (${}^*F_{ab} = \pm iF_{ab}$), which are also extrema for the action functional:

$$S = \int F_{ab} \wedge {}^*F^{ab}\, dV$$

The significance of α is thus emerging both from a physical (extremal) and from a geometrical principle (bundle structure).

The situation is rather similar for the gravitational field. We briefly summarize results obtained in Magnon (1986, 1988, 1989) and references therein. Focusing on Maxwellian aspects of the gravitational field, and assuming the existence of a conformal isometry ξ ($\mathscr{L}_\xi g_{ab} = \chi g_{ab}$), we can introduce two conserved (stress-energy) tensors ${}^*K_{ab}$ and K_{ab}, respectively the magnetic and electric components of the spin-2 field K_{abcd} (rescaled Weyl tensor), w.r.t. the asymptotic null observer $\partial/\partial u$ (limit of ξ in the null directions). Gravitational analogues of the electric and magnetic charges can be defined:

(a) $\int_{S_2} l^a\, {}^*K_{ab}\varepsilon^{\ b}_{mn}\, dS^{mn}$, the total mass
(b) $\int_{S_2} l^n\, K_{ab}\varepsilon^{\ b}_{mn}\, dS^{mn}$, the total magnetic mass

where $l \cdot n = -1$, and S_2 is the manifold of orbits of n^a (the null generators $\partial/\partial u$). Due to breaking of the usual notion of asymptotic flatness at spacelike infinity if the magnetic mass does not vanish, the asymptotic space-time structure is that of a nontrivial $U(1)$ bundle over S_2 with horizontal cross sections locally defined by the bundle connection 1-form $\mathscr{A}_b$. This form is the lift of a 1-form a_b on S_2, and a_b displays a discontinuity describing the impact, at infinity, of the gravitational analogue of a Dirac string (which we call gravitational string). The ratio of the magnetic mass to the mass is quantized [number of windings of the bundle around its $U(1)$ fiber], and an integer (n) multiple of β, the gravitational analogue of the fine structure constant α.

Again, the existence of β as a proportionality factor is guaranteed, in the complex regime, by the existence (Magnon, 1986) of self and antiself adjoint gauge connections $\mathscr{L}_b$, which extremize the (asymptotic) action functional:

$$\mathscr{S}=\int_{S_2} \mathfrak{T}_{ab} \wedge {}^*\mathfrak{T}^{ab}\, dV=\int_{S_2} (\varepsilon_{abp}\, {}^*K^{pm} l_m)(\varepsilon^{abq} K_{qn} l^n)\, dV$$

Finally, in this setup, the expanding space-time is acausal [$U(1)$ bundle], particle production is locally taking place along the (nonstationary) ξ^a orbits; however, matter–antimatter cannot be globally decoupled due to acausality. This can be viewed as a manifestation of the quantum vacuum stabilized by the above action principle and coupling constants such as α or β. In this context spontaneous creation can occur but global matter–antimatter decoupling is prevented by the existence of strings (gravitational or Maxwellian) and acausality. Dissipation of such strings, in relation with acausality breaking, will be investigated in the next section.

4. ACAUSALITY BREAKING AND GRAVITATIONAL INTERACTION

In this section, and in conclusion, we would like to raise the following question. Can one manufacture a mechanism by which, not only could matter–antimatter decoupling take place, but, and by the same token, gravitational interaction could be switched on as a causal phenomenon?

We propose here to relate this issue to a mechanism of acausality breaking based on string dissipation. Recall that, in Section 2, onset of curvature via quantum corrections to the space-time metric also provided a mechanism for the onset of horizons, causal asymptotic (mirror) regions, and matter–antimatter decoupling. The main objection was that field equations were not taken into consideration for the induced curvature.

In Section 3, asymptotic field equations (via conserved tensors K_{ab} and ${}^*K_{ab}$, coupling constants, and extrema of the action) were brought into consideration in a context where pair creation could in principle take place, but matter–antimatter decoupling could not globally take place due to acausality.

We propose now to gather these results into a scheme of switching on of the gravitational interaction based on the onset of causality and dissipation of gravitational strings.

Recall that Newton's gravitational law is based on the identification of the gravitational and the inertial masses:

$$m_g/m_i = \text{universal constant}$$

Recall that, in the presence of gravitational strings, we have shown that asymptotic field equations and extrema of the action are correlated to the proportionality of the mass M and magnetic mass M^*:

$$M/M^* = \text{universal constant}$$

Recall also that, in the presence of gravitational strings ($M^* \neq 0$), acausality cannot be removed by going to a covering space since the asymptotic structure is that of a lens space [nontrivial $U(1)$ Hopf fibration over S_2, with n identifications along the fiber] whose covering space is S^3.

An identification of M^* with m_i could thus fix the gravitational coupling strength provided it can be related to a physical mechanism by which acausality would disappear.

We propose now such a mechanism. We first recall the existence of a mathematical transformation by which magnetic mass solutions to Einstein's vacuum equation can be converted into black-hole solutions:

$$M^* \to M$$

We shall then show that this transformation can (and at the Planck scale) be correlated to a physical principle based on inertial decoupling of matter from antimatter, and onset of a causal gravitational coupling (curvature):

$$M_i \to M$$

Recall first that a stationary solution to Einstein's vacuum equation can be characterized by a complex potential τ on the manifold Σ of orbits of the stationary Killing field t^a:

$$\tau = \omega + i\lambda$$

(λ and ω are, respectively, the norm and twist of t^a). The solution is also characterized by potentials on Σ:

$$\phi_M = \tfrac{1}{4}(\lambda^2 + \omega^2 - 1), \qquad \text{the mass monopole}$$

$$\phi_j = \omega/2\lambda, \qquad \text{the dual mass monopole}$$

For the Schwarzschild black-hole solution, ϕ_M is nonzero ($=M$) and ϕ_J vanishes. For the NUT magnetic monopole solution, $\phi'_M = 0$ and $\phi'_J = -\phi_M$. These solutions are related by the Geroch mathematical transformation (duality rotation):

$$\begin{pmatrix} \phi'_M \\ \phi'_J \end{pmatrix} = \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{pmatrix} \begin{pmatrix} \phi_M \\ \phi_J \end{pmatrix}$$

for $\theta=\pi/4$. Introducing the spacelike 2-plane Π generated by $\mathbf{v}_1=\text{grad }\phi_M$ and $\mathbf{v}_2=\text{grad }\phi_J$, at each point of Σ, we will show elsewhere (Boudet and Magnon, 1991) that this transformation rotates Π according to

$$\begin{pmatrix}\mathbf{v}_1'\\ \mathbf{v}_2'\end{pmatrix}=\begin{pmatrix}\cos 2\theta & -\sin 2\theta\\ \sin 2\theta & \cos 2\theta\end{pmatrix}\begin{pmatrix}\mathbf{v}_1\\ \mathbf{v}_2\end{pmatrix}$$

inducing a gauge transformation on

$$A_b=\mathbf{v}_1\cdot\partial_b\mathbf{v}_2$$

$$A_b'=A_b+e\,\partial_b 2\theta$$

The same holds true for $\hat{A}_b$, the potential of $\hat{F}_{ab}=-\lambda^{-3/2}(D_m\omega)\in{}^m{}_{ab}$, curl-free 2-form on Σ whose flux measures M^* in the presence of noncontractible 2-spheres (recall that F_{ab} is the lift of $\hat{F}_{ab}$: $F_{ab}=\nabla_{[a}\lambda^{-1}t_{b]}$). This (duality) transformation thus induces a reshuffling of the Killing horizon into a singularity of the gauge potential (at the origin of Π); and in particular transformation of the Schwarzschild horizon ($\lambda=0$) into the Taub–Nut wire singularity.

One is thus entitled to search for a physical process to support this mathematical transformation.

Recall from Section 2 that the onset of a horizon can be imputed to a quantum and inertial correction of a flat space-time metric:

$$\lambda=-1+\frac{2M}{r}\simeq-1+\frac{M_i^2\hbar^2}{\alpha^2}$$

In this setup, the mathematical transformation $\phi_J'\to\phi_M$ (to be viewed as a conversion of inertial magnetic mass into mass) could thus be supported by the following physical scheme: pair creation along nongeodesic accelerated trajectories, quantum correction to the metric, onset of a (Killing) horizon and related causal asymptotic regions, and finally matter–antimatter separation after dissipation of a gravitational string and breaking of the $U(1)$ acausality.

These considerations valid in the context of Planckian mini-black-holes could be generalized to broader scales, including those compatible with the expansion field of an FRW universe. They can also be viewed as relating the (classical) principle of equal accelerations (equivalence principle) to quantum considerations. We hope they shed some light on the definition of the gravitational interaction at the Planck scale and possible emergence of our causal expanding universe from quantum processes such as dissipation of strings in the decoupling of matter from antimatter.

ACKNOWLEDGMENT

Thanks are due to Prof. Roger Boudet (Marseille University, France) for stimulating discussions.

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